Modeling of Two-Phase Flow for Estimation and Control of Drilling Operations

Ulf Jakob F. Aarsnes

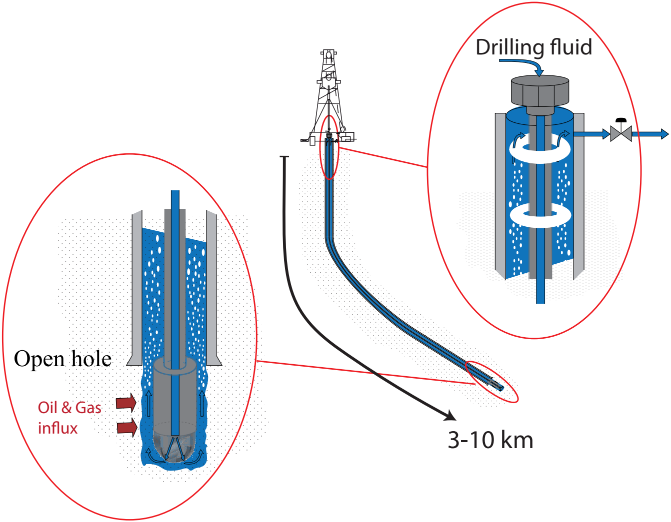
Department of Engineering Cybernetics, NTNU
Supervisor: Prof. Ole Morten Aamo
Co-supervisor: Dr. Glenn-Ole Kaasa

May 24, 2016

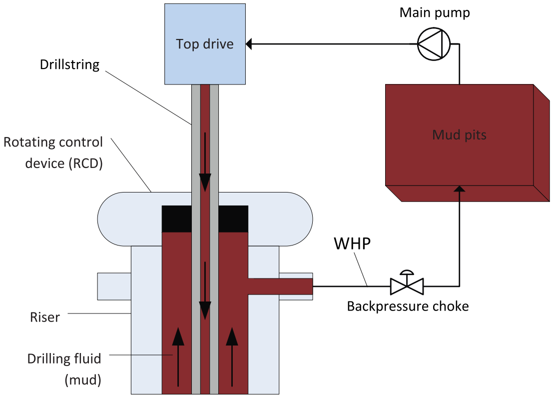
Pressure control in drilling

Pressure is controlled from the top.

Open-hole pressure must be kept within constraints.



Topside schematic



Pressure control is achieved by a combination of:

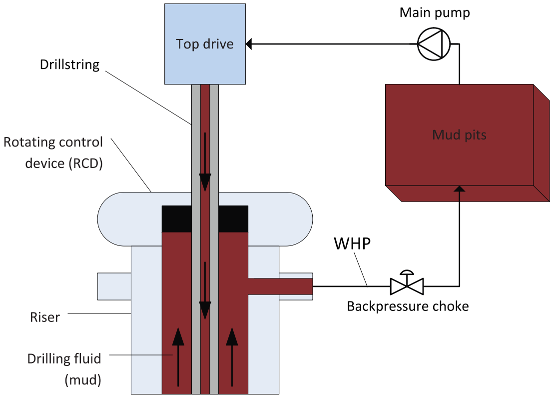
- Backpressure (WHP)
- Hydrostatic Pressure
- Frictional pressure

At steady state:

Bottomhole pressure

$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Topside schematic



Pressure control is achieved by a combination of:

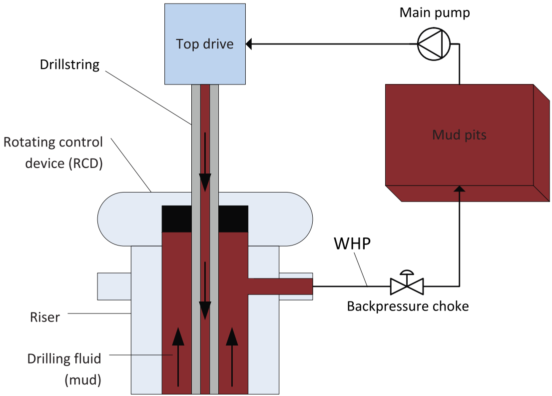
- Backpressure (WHP)
- Hydrostatic Pressure
- Frictional pressure

At steady state:

Bottomhole pressure

$$\text{BHCP} = \text{WHP} + G + F$$

Topside schematic



Pressure control is achieved by a combination of:

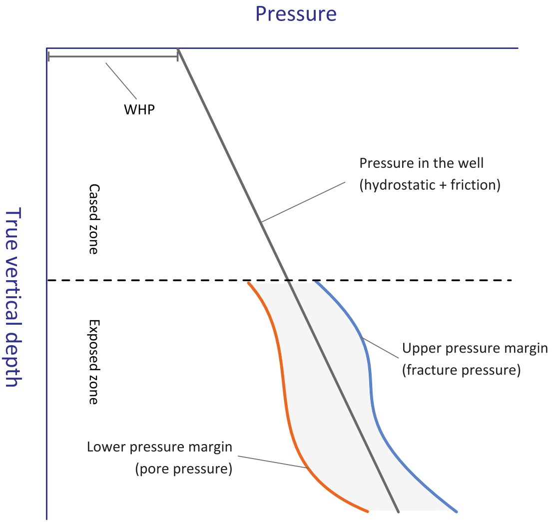
- Backpressure (WHP)
- Hydrostatic Pressure
- Frictional pressure

At steady state:

Bottomhole pressure

$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Pressure control in MPD



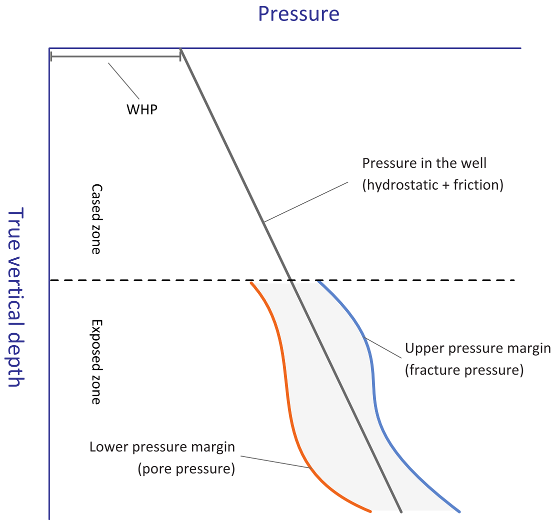
At steady state:

$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Control goal (simplified):

$$P_{pore} < BHCP < P_{frac}$$

Pressure control in MPD



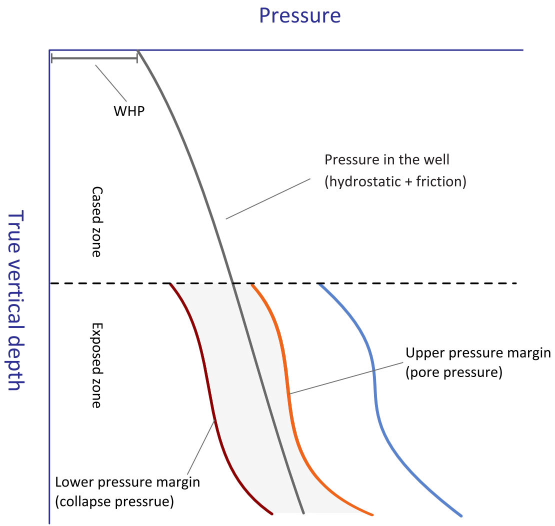
At steady state:

$$\text{BHCP} = \text{WHP} + G + F$$

Control goal (simplified):

$$P_{pore} < BHCP < P_{frac}$$

Pressure control in UBD



At steady state:

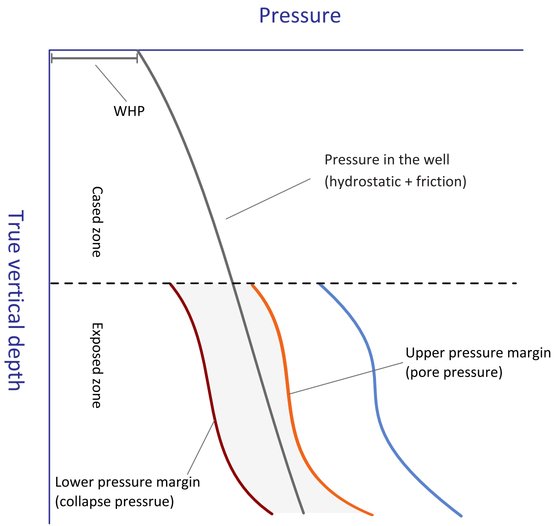
$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Control goal (simplified):

$$P_{collapse} < BHCP < P_{pore}$$

- 2-phase dynamics
- Reservoir interaction

Pressure control in UBD



At steady state:

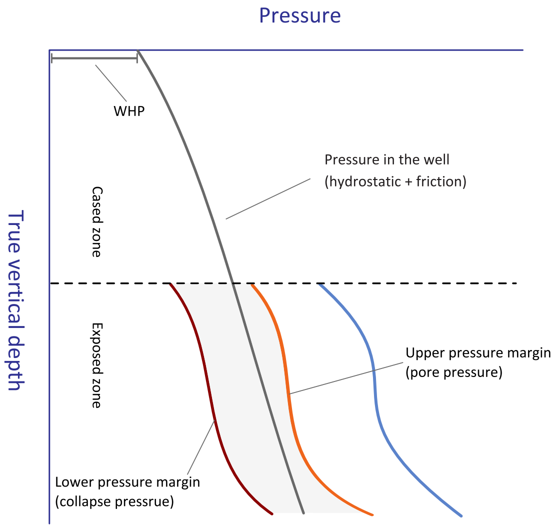
$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Control goal (simplified):

$$P_{collapse} < BHCP < P_{pore}$$

- 2-phase dynamics
- Reservoir interaction

Pressure control in UBD



At steady state:

$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

Control goal (simplified):

$$P_{collapse} < BHCP < P_{pore}$$

- 2-phase dynamics
- Reservoir interaction

Research objective

Simple mathematical models

- Allows the use of more advanced mathematical tools
- Eases implementation and application of results
- Understandable behavior $\rightarrow$ robust algorithms

Research objective

Two-phase flow modeling for Estimation and Control

- Find the right compromise between model complexity and model fidelity.
- Develop fit-for-purpose models.

Approach

Analyze system behavior to find the dominating dynamics to be represented for a given application and timescale.

Research objective

Simple mathematical models

- Allows the use of more advanced mathematical tools
- Eases implementation and application of results
- Understandable behavior $\rightarrow$ robust algorithms

Research objective

Two-phase flow modeling for Estimation and Control

- Find the right compromise between model complexity and model fidelity.
- Develop fit-for-purpose models.

Approach

Analyze system behavior to find the dominating dynamics to be represented for a given application and timescale.

Research objective

Simple mathematical models

- Allows the use of more advanced mathematical tools
- Eases implementation and application of results
- Understandable behavior $\rightarrow$ robust algorithms

Research objective

Two-phase flow modeling for Estimation and Control

- Find the right compromise between model complexity and model fidelity.
- Develop fit-for-purpose models.

Approach

Analyze system behavior to find the dominating dynamics to be represented for a given application and timescale.

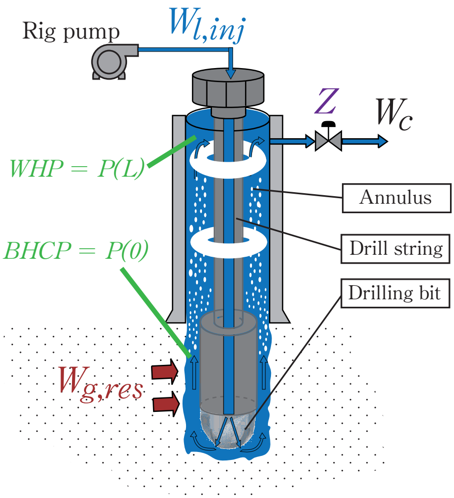
Control and Estimation challenges

From practical need to control problem

- Characterize operating conditions $\rightarrow$ Linear stability analysis
- Respect pressure constraints $\rightarrow$ Disturbance rejection and tracking
- Monitor gas quantity in the pipe $\rightarrow$ State estimation
- Estimation of reservoir characteristics $\rightarrow$ Parameter identification

Many important challenges that can be addressed by modern control techniques.

System under consideration



- Downhole pressure function of gas amount and flow:

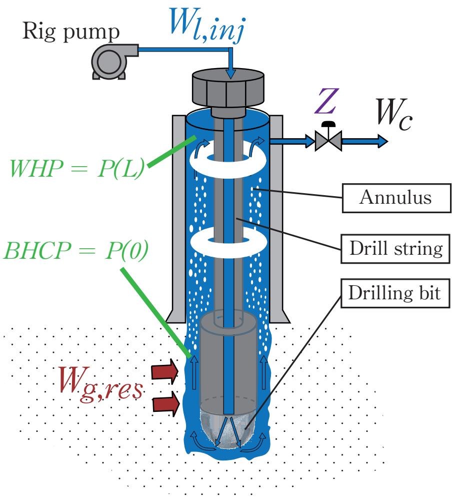
$$\text{BHCP} = \text{WHP} + G + F$$

- Gas influx function of downhole pressure

$$W_{G,res} = \text{IPR}(P_{res} - \text{BHCP})$$

- Feedback loop.

System under consideration



- Downhole pressure function of gas amount and flow:

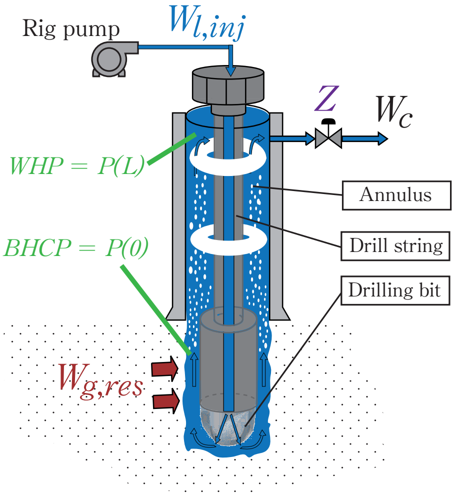
$$\mathrm{BHCP} = \mathrm{WHP} + G + F$$

- Gas influx function of downhole pressure

$$W_{G,res} = \mathrm{IPR}(P_{res} - \mathrm{BHCP})$$

- Feedback loop.

System under consideration



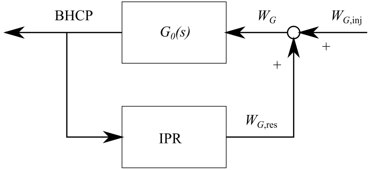
- Downhole pressure function of gas amount and flow:

$$\text{BHCP} = \text{WHP} + G + F$$

- Gas influx function of downhole pressure

$$W_{G,res} = \text{IPR}(P_{res} - \text{BHCP})$$

- Feedback loop.



Steady State solutions

Drift Flux Model (Lage et al., 2000)

$$\text{WHP} = \text{BHCP} + \int_0^L -\underbrace{\frac{\partial m v_L^2 + n v_G^2}{\partial s}}_{\text{Acceleration}} - \underbrace{(m+n) g \sin\phi(s)}_{\text{Gravity}} - \underbrace{\frac{2f(m+n)v_m|v_m|}{D}}_{\text{Friction}} \,\mathrm{d}s,$$

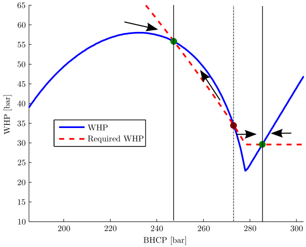
$$\left.\begin{aligned} Amv_L &= k_L \max(P_{res} - \text{BHCP}, 0) + W_{L,inj}(t), \\ Anv_G &= k_G \max(P_{res} - \text{BHCP}, 0) + W_{G,inj}(t), \end{aligned}\right\} = \text{Boundary conditions}$$

Steady State solutions

Drift Flux Model (Lage et al., 2000)

$$\mathrm{WHP} = \mathrm{BHCP} + \int_0^L -\underbrace{\frac{\partial m v_L^2 + n v_G^2}{\partial s}}_{\text{Acceleration}} - \underbrace{(m+n) g \sin\phi(s)}_{\text{Gravity}} - \underbrace{\frac{2f(m+n)v_m|v_m|}{D}}_{\text{Friction}} \, \mathrm{d}s,$$

$$\left.\begin{aligned} A m v_L &= k_L \max(P_{res} - \mathrm{BHCP}, 0) + W_{L,inj}(t), \\ A n v_G &= k_G \max(P_{res} - \mathrm{BHCP}, 0) + W_{G,inj}(t), \end{aligned}\right\} = \text{Boundary conditions}$$

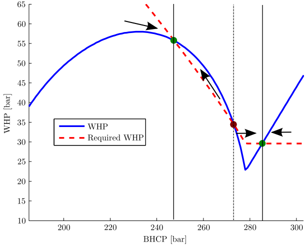


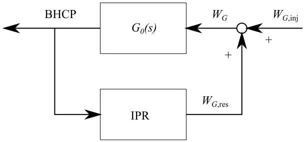
Steady State solutions

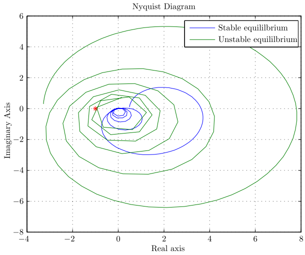
Drift Flux Model (Lage et al., 2000)

$$\text{WHP} = \text{BHCP} + \int_0^L - \underbrace{\frac{\partial m v_L^2 + n v_G^2}{\partial s}}_{\text{Acceleration}} - \underbrace{(m+n) g \sin\phi(s)}_{\text{Gravity}} - \underbrace{\frac{2f(m+n)v_m|v_m|}{D}}_{\text{Friction}} \, \mathrm{d}s,$$

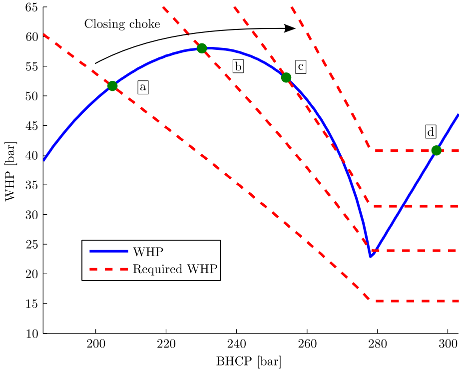
$$\left.\begin{aligned} A m v_L &= k_L \, \max(P_{res} - \text{BHCP}, 0) + W_{L,inj}(t), \\ A n v_G &= k_G \, \max(P_{res} - \text{BHCP}, 0) + W_{G,inj}(t), \end{aligned}\right\} = \text{Boundary conditions}$$

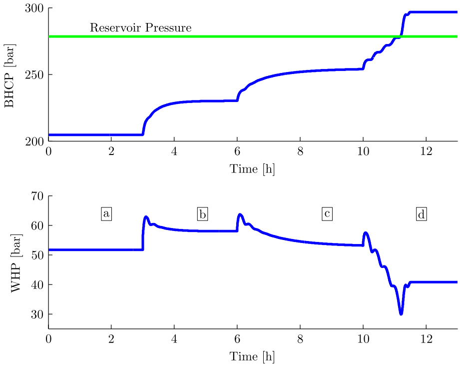






Transient simulation

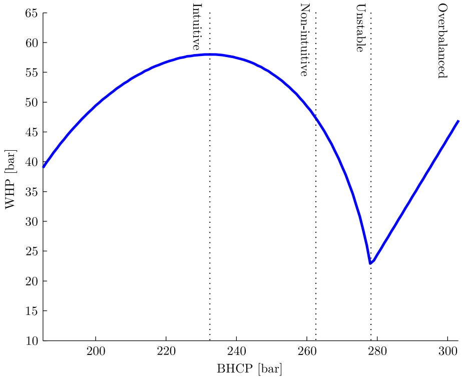




Asymptotic behaviour:

- Red line above blue: move right.
- Red line below blue: move left.

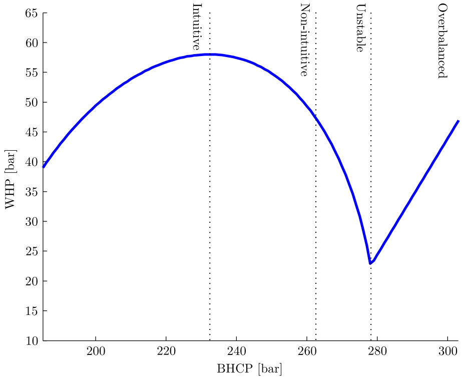
Characterising Dynamics



Classification of operating regimes

- Intuitive
  BHCP changes in same direction as WHP.
- Non-Intuitive
  Inverse response and rapidly changing dynamics.
- Unstable
  The well is open-loop unstable.
- Overbalanced
  One-phase dynamics.

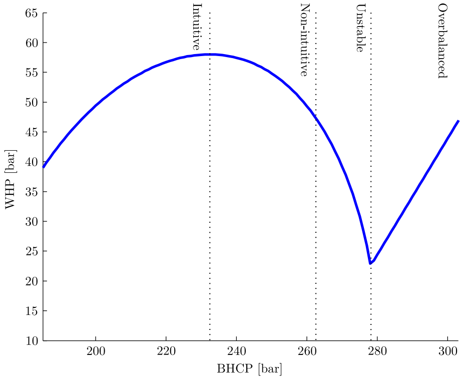
Characterising Dynamics



Classification of operating regimes

- **Intuitive**
  BHCP changes in same direction as WHP.
- **Non-Intuitive**
  Inverse response and rapidly changing dynamics.
- **Unstable**
  The well is open-loop unstable.
- **Overbalanced**
  One-phase dynamics.

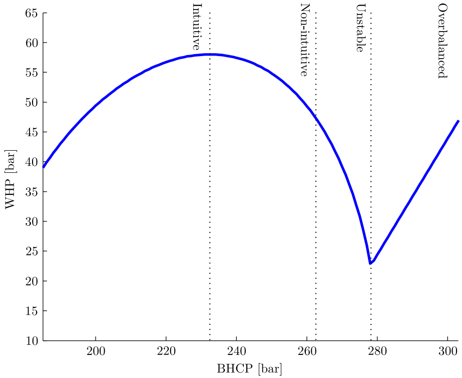
Characterising Dynamics



Classification of operating regimes

- **Intuitive**
  BHCP changes in same direction as WHP.
- **Non-Intuitive**
  Inverse response and rapidly changing dynamics.
- **Unstable**
  The well is open-loop unstable.
- **Overbalanced**
  One-phase dynamics.

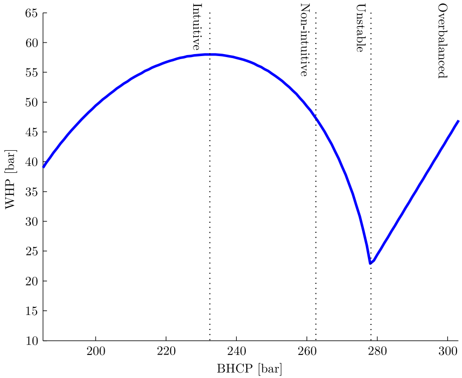
Characterising Dynamics



Classification of operating regimes

- Intuitive
  BHCP changes in same direction as WHP.
- Non-Intuitive
  Inverse response and rapidly changing dynamics.
- Unstable
  The well is open-loop unstable.
- Overbalanced
  One-phase dynamics.

Characterising Dynamics

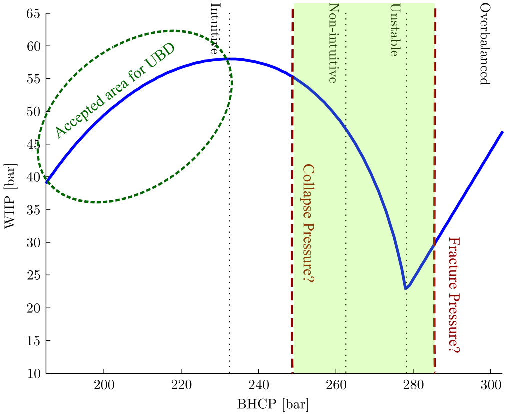


Classification of operating regimes

- Intuitive
  BHCP changes in same direction as WHP.
- Non-Intuitive
  Inverse response and rapidly changing dynamics.
- Unstable
  The well is open-loop unstable.
- Overbalanced
  One-phase dynamics.

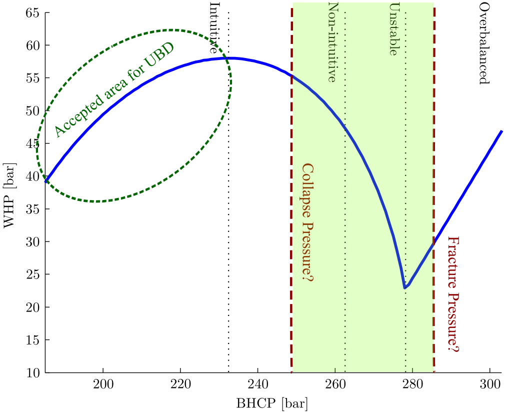
At balance/Low-Drawdown drilling

- Underbalanced drilling entails significant benefits.
- A major obstacle to UBD is limitations on allowable drawdown.
- Automatic control could stabilize the well at a low drawdown.



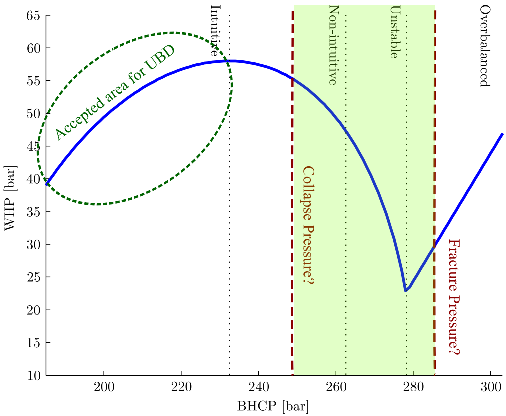
At balance/Low-Drawdown drilling

- Underbalanced drilling entails significant benefits.
- A major obstacle to UBD is limitations on allowable drawdown.
- Automatic control could stabilize the well at a low drawdown.



At balance/Low-Drawdown drilling

- Underbalanced drilling entails significant benefits.
- A major obstacle to UBD is limitations on allowable drawdown.
- Automatic control could stabilize the well at a low drawdown.



- The Drift Flux Model is the most used model for two-phase flow in drilling.
  - Drift Flux Model, however, not most general model.
- Most general one-dimensional two-phase formulation: Baer-Nunziato.
  - Too complicated for many applications.

- The Drift Flux Model is the most used model for two-phase flow in drilling.
  - Drift Flux Model, however, not most general model.
- Most general one-dimensional two-phase formulation: Baer-Nunziato.
  - Too complicated for many applications.

The Baer Nunziato Formulation (Baer and Nunziato, 1986)

For two phases: liquid ℓ and gas g:

Volume advection:

$$\frac{\partial \alpha_\mathrm{g}}{\partial t} + v_p \frac{\partial \alpha_\mathrm{g}}{\partial x} = \mathcal{J}(P_\mathrm{g} - P_\ell), \tag{1}$$

Mass conservation:

$$\frac{\partial}{\partial t}\left(\rho_\mathrm{g}\alpha_\mathrm{g}\right) + \frac{\partial}{\partial x}\left(\rho_\mathrm{g}\alpha_\mathrm{g} v_\mathrm{g}\right) = \mathcal{K}(\mu_\ell - \mu_\mathrm{g}), \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho_\ell\alpha_\ell\right) + \frac{\partial}{\partial x}\left(\rho_\ell\alpha_\ell v_\ell\right) = \mathcal{K}(\mu_\mathrm{g} - \mu_\ell), \tag{3}$$

Momentum balance:

$$\frac{\partial}{\partial t}\left(\rho_\mathrm{g}\alpha_\mathrm{g} v_\mathrm{g}\right) + \frac{\partial}{\partial x}\left(\rho_\mathrm{g}\alpha_\mathrm{g} v_\mathrm{g}^2 + \alpha_\mathrm{g} P_\mathrm{g}\right) - p_\mathrm{i}\frac{\partial \alpha_\mathrm{g}}{\partial x} = v_\mathrm{i}\mathcal{K}(\mu_\ell - \mu_\mathrm{g}) + \mathcal{M}(v_\ell - v_\mathrm{g}), \tag{4}$$

$$\frac{\partial}{\partial t}\left(\rho_\ell\alpha_\ell v_\ell\right) + \frac{\partial}{\partial x}\left(\rho_\ell\alpha_\ell v_\ell^2 + \alpha_\ell P_\ell\right) + p_\mathrm{i}\frac{\partial \alpha_\mathrm{g}}{\partial x} = v_\mathrm{i}\mathcal{K}(\mu_\mathrm{g} - \mu_\ell) + \mathcal{M}(v_\mathrm{g} - v_\ell), \tag{5}$$

Energy balance:

$$\begin{aligned}\frac{\partial E_\mathrm{g}}{\partial t} + \frac{\partial}{\partial x}\left(E_\mathrm{g} v_\mathrm{g} + \alpha_\mathrm{g} P_\mathrm{g} v_\mathrm{g}\right) - p_\mathrm{i} v_p \frac{\partial \alpha_\mathrm{g}}{\partial x} &= -p_\mathrm{i}\mathcal{J}(P_\ell - P_\mathrm{g}) \\ &+ \left(\mu_\mathrm{i} + \frac{1}{2}v_\mathrm{i}^2\right)\mathcal{K}(\mu_\ell - \mu_\mathrm{g}) + v_p\mathcal{M}(v_\ell - v_\mathrm{g}) + \mathcal{H}(T_\ell - T_\mathrm{g}),\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial E_\ell}{\partial t} + \frac{\partial}{\partial x}\left(E_\ell v_\ell + \alpha_\ell P_\ell v_\ell\right) + p_\mathrm{i} v_p \frac{\partial \alpha_\mathrm{g}}{\partial x} &= -p_\mathrm{i}\mathcal{J}(P_\mathrm{g} - P_\ell) \\ &+ \left(\mu_\mathrm{i} + \frac{1}{2}v_\mathrm{i}^2\right)\mathcal{K}(\mu_\mathrm{g} - \mu_\ell) + v_p\mathcal{M}(v_\mathrm{g} - v_\ell) + \mathcal{H}(T_\mathrm{g} - T_\ell).\end{aligned} \tag{7}$$

Hierarchy of relaxation models (Linga, 2016)

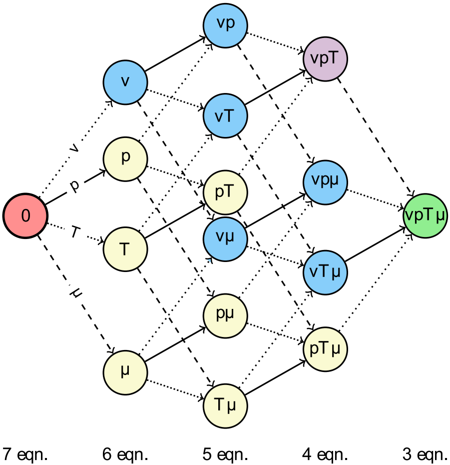


Figure: Hypercube representing hierarchy of 2-phase relaxation models. Edges are relaxation process' removing an equation.

Hierarchy of relaxation models (Linga, 2016)

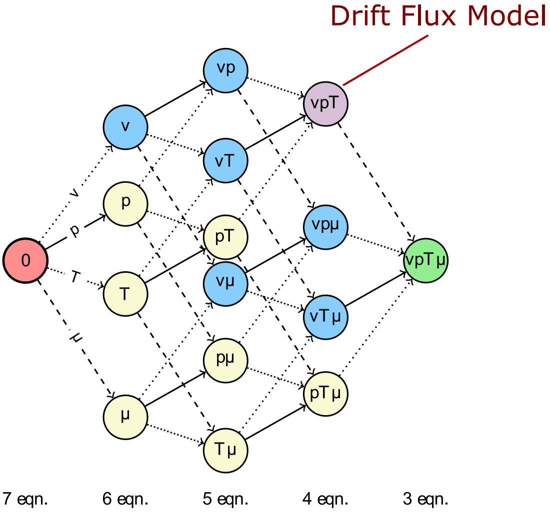


Figure: Hypercube representing hierarchy of 2-phase relaxation models. Edges are relaxation process' removing an equation.

Dynamic Drift-Flux Model (DFM)
(Zuber and Findlay, 1965; Evje and Wen, 2015)

Mass & momentum conservation laws

Mass of gas: $$\frac{\partial \alpha_\ell \rho_\ell}{\partial t} + \frac{\partial \alpha_\ell \rho_\ell v_\ell}{\partial x} = 0$$

Mass of Liquid: $$\frac{\partial \alpha_g \rho_g}{\partial t} + \frac{\partial \alpha_g \rho_g v_g}{\partial x} = 0$$

Combined Momentum Equation:

$$\frac{\partial \alpha_\ell \rho_\ell v_\ell + \alpha_g \rho_g v_g}{\partial t} + \frac{\partial P + \alpha_\ell \rho_\ell v_\ell^2 + \alpha_g \rho_g v_g^2}{\partial x} = S,$$

Closure relation

$$v_g = C_0 v_M + v_\infty, \quad P = c_g^2 \rho_g$$

Dynamic Drift-Flux Model (DFM)
(Zuber and Findlay, 1965; Evje and Wen, 2015)

Mass & momentum conservation laws

$$\textbf{Mass of gas:} \quad \frac{\partial \alpha_\ell \rho_\ell}{\partial t} + \frac{\partial \alpha_\ell \rho_\ell v_\ell}{\partial x} = 0$$

$$\textbf{Mass of Liquid:} \quad \frac{\partial \alpha_\mathrm{g} \rho_\mathrm{g}}{\partial t} + \frac{\partial \alpha_\mathrm{g} \rho_\mathrm{g} v_\mathrm{g}}{\partial x} = 0$$

Combined Momentum Equation:

$$\frac{\partial \alpha_\ell \rho_\ell v_\ell + \alpha_\mathrm{g} \rho_\mathrm{g} v_\mathrm{g}}{\partial t} + \frac{\partial P + \alpha_\ell \rho_\ell v_\ell^2 + \alpha_\mathrm{g} \rho_\mathrm{g} v_\mathrm{g}^2}{\partial x} = S,$$

Closure relation

$$v_\mathrm{g} = C_0 v_M + v_\infty, \quad P = c_\mathrm{g}^2 \rho_\mathrm{g}$$

Characteristics of Hyperbolic systems

In quasilinear form:

$$\frac{\partial q}{\partial t} + A(q)\frac{\partial q}{\partial x} = G(q)$$

Transport velocities:

- $A(q)$: 3×3 matrix with eigenvectors $\lambda_1, \lambda_2, \lambda_3$
- $\lambda_1 = v_G \approx 1 - 10$ m.s^{-1}: liquid & gas (void wave) transport
- $\lambda_2 \approx -\lambda_3 \approx c_M \approx 100 - 1000$ m.s^{-1}: pressure waves

Characteristics of Hyperbolic systems

In quasilinear form:

$$\frac{\partial q}{\partial t} + A(q)\frac{\partial q}{\partial x} = G(q)$$

Transport velocities:

- $A(q)$: 3×3 matrix with eigenvectors $\lambda_1, \lambda_2, \lambda_3$
- $\lambda_1 = v_G \approx 1 - 10$ m.s^{-1}: liquid & gas (void wave) transport
- $\lambda_2 \approx -\lambda_3 \approx c_M \approx 100 - 1000$ m.s^{-1}: pressure waves

Characteristics of Hyperbolic systems

In quasilinear form:

$$\frac{\partial q}{\partial t} + A(q)\frac{\partial q}{\partial x} = G(q)$$

Transport velocities:

- $A(q)$: 3×3 matrix with eigenvectors $\lambda_1, \lambda_2, \lambda_3$
- $\lambda_1 = v_G \approx 1 - 10$ m.s^{-1}: liquid & gas (void wave) transport
- $\lambda_2 \approx -\lambda_3 \approx c_M \approx 100 - 1000$ m.s^{-1}: pressure waves

Possible to decompose system into fast and slow dynamics.

Approximation

Transformation due to Gavrilyuk and Fabre (1996)

$$\mathbf{u} = (\chi_\ell, \rho, v_\mathrm{g}) = \left(\frac{(\alpha_\ell - \alpha_\ell^*)\rho_\ell}{\rho_M - \alpha_\ell^* \rho_\ell}, \rho_M - \alpha_\ell^* \rho_\ell, v_\mathrm{g} \right),$$

to obtain equivalent system (approximation):

$$\frac{\partial}{\partial t}\begin{bmatrix}\chi_\ell \\ \rho \\ v_\mathrm{g}\end{bmatrix} + \begin{bmatrix} v_\mathrm{g} & 0 & 0 \\ 0 & v_\mathrm{g} & \rho \\ \frac{\bar{\alpha}_0(\mathbf{u}) c_M^2(\mathbf{u})}{\rho} & \frac{c_M^2(\mathbf{u})}{\rho} & v_\mathrm{g} \end{bmatrix} \frac{\partial}{\partial x}\begin{bmatrix}\chi_\ell \\ \rho \\ v_\mathrm{g}\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ \tilde{S}\end{bmatrix},$$

We note that:

- For constants mass rates W_g, W_ℓ, the psuedo hold-up is $\chi_\ell = const$.
- Relatively weak coupling to velocity and density dynamics v_g, ρ.
- Tempting to "diagonalize" the system.

Approximation

Transformation due to Gavrilyuk and Fabre (1996)

$$\mathbf{u} = (\chi_\ell, \rho, v_\mathrm{g}) = \left(\frac{(\alpha_\ell - \alpha_\ell^*)\rho_\ell}{\rho_M - \alpha_\ell^* \rho_\ell}, \rho_M - \alpha_\ell^* \rho_\ell, v_\mathrm{g} \right),$$

to obtain equivalent system (approximation):

$$\frac{\partial}{\partial t} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} + \begin{bmatrix} v_\mathrm{g} & 0 & 0 \\ 0 & v_\mathrm{g} & \rho \\ \frac{\bar{\alpha}_0(\mathbf{u}) c_M^2(\mathbf{u})}{\rho} & \frac{c_M^2(\mathbf{u})}{\rho} & v_\mathrm{g} \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{S} \end{bmatrix},$$

We note that:

- For constants mass rates W_g, W_ℓ, the psuedo hold-up is $\chi_\ell = const$.
- Relatively weak coupling to velocity and density dynamics v_g, ρ.
- Tempting to "diagonalize" the system.

Approximation

Transformation due to Gavrilyuk and Fabre (1996)

$$\mathbf{u} = (\chi_\ell, \rho, v_\mathrm{g}) = \left(\frac{(\alpha_\ell - \alpha_\ell^*)\rho_\ell}{\rho_M - \alpha_\ell^* \rho_\ell}, \rho_M - \alpha_\ell^* \rho_\ell, v_\mathrm{g} \right),$$

to obtain equivalent system (approximation):

$$\frac{\partial}{\partial t} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} + \begin{bmatrix} v_\mathrm{g} & 0 & 0 \\ 0 & v_\mathrm{g} & \rho \\ \frac{\bar{\alpha}_0(\mathbf{u}) c_M^2(\mathbf{u})}{\rho} & \frac{c_M^2(\mathbf{u})}{\rho} & v_\mathrm{g} \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{S} \end{bmatrix},$$

We note that:

- For constants mass rates W_g, W_ℓ, the psuedo hold-up is $\chi_\ell = const$.
- Relatively weak coupling to velocity and density dynamics v_g, ρ.
- Tempting to "diagonalize" the system.

Approximation

Transformation due to Gavrilyuk and Fabre (1996)

$$\mathbf{u} = (\chi_\ell, \rho, v_\mathrm{g}) = \left(\frac{(\alpha_\ell - \alpha_\ell^*)\rho_\ell}{\rho_M - \alpha_\ell^* \rho_\ell}, \rho_M - \alpha_\ell^* \rho_\ell, v_\mathrm{g} \right),$$

to obtain equivalent system (approximation):

$$\frac{\partial}{\partial t} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} + \begin{bmatrix} v_\mathrm{g} & 0 & 0 \\ 0 & v_\mathrm{g} & \rho \\ \frac{\bar{\alpha}_0(\mathbf{u}) c_M^2(\mathbf{u})}{\rho} & \frac{c_M^2(\mathbf{u})}{\rho} & v_\mathrm{g} \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \chi_\ell \\ \rho \\ v_\mathrm{g} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{S} \end{bmatrix},$$

We note that:

- For constants mass rates W_g, W_ℓ, the psuedo hold-up is $\chi_\ell = const$.
- Relatively weak coupling to velocity and density dynamics v_g, ρ.
- Tempting to "diagonalize" the system.

System diagonalization

- The transformed mass variable χ_ℓ dynamics independent w.r.t. rest of system.
- For constant mass-rates at the left boundary, $\chi_\ell = const$.
- Then the distributed pressure dynamics become:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} + \begin{bmatrix} v_{\mathrm{g}} & \rho \\ \frac{c_M^2(\mathbf{u})}{\rho} & v_{\mathrm{g}}\end{bmatrix}\frac{\partial}{\partial x}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} = \begin{bmatrix}0 \\ \tilde{S}\end{bmatrix}, \begin{bmatrix}\lambda_{s1} \\ \lambda_{s2}\end{bmatrix} = \begin{bmatrix} v_{\mathrm{g}} + c_M(\mathbf{u}) \\ v_{\mathrm{g}} - c_M(\mathbf{u})\end{bmatrix}$$

- Equivalent to well known wave equation for $c_M(\mathbf{u}) \gg v_{\mathrm{g}}$.

System diagonalization

- The transformed mass variable χ_ℓ dynamics independent w.r.t. rest of system.
- For constant mass-rates at the left boundary, $\chi_\ell = const$.
- Then the distributed pressure dynamics become:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ v_\mathrm{g}\end{bmatrix} + \begin{bmatrix} v_\mathrm{g} & \rho \\ \frac{c_M^2(\mathbf{u})}{\rho} & v_\mathrm{g}\end{bmatrix}\frac{\partial}{\partial x}\begin{bmatrix}\rho \\ v_\mathrm{g}\end{bmatrix} = \begin{bmatrix}0 \\ \tilde{S}\end{bmatrix}, \begin{bmatrix}\lambda_{s1} \\ \lambda_{s2}\end{bmatrix} = \begin{bmatrix} v_\mathrm{g} + c_M(\mathbf{u}) \\ v_\mathrm{g} - c_M(\mathbf{u})\end{bmatrix}$$

- Equivalent to well known wave equation for $c_M(\mathbf{u}) \gg v_\mathrm{g}$.

System diagonalization

- The transformed mass variable χ_ℓ dynamics independent w.r.t. rest of system.
- For constant mass-rates at the left boundary, $\chi_\ell = const$.
- Then the distributed pressure dynamics become:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} + \begin{bmatrix} v_{\mathrm{g}} & \rho \\ \frac{c_M^2(\mathbf{u})}{\rho} & v_{\mathrm{g}}\end{bmatrix}\frac{\partial}{\partial x}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} = \begin{bmatrix}0 \\ \tilde{S}\end{bmatrix}, \begin{bmatrix}\lambda_{s1} \\ \lambda_{s2}\end{bmatrix} = \begin{bmatrix} v_{\mathrm{g}} + c_M(\mathbf{u}) \\ v_{\mathrm{g}} - c_M(\mathbf{u})\end{bmatrix}$$

- Equivalent to well known wave equation for $c_M(\mathbf{u}) \gg v_{\mathrm{g}}$.

System diagonalization

- The transformed mass variable χ_ℓ dynamics independent w.r.t. rest of system.
- For constant mass-rates at the left boundary, $\chi_\ell = const$.
- Then the distributed pressure dynamics become:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} + \begin{bmatrix} v_{\mathrm{g}} & \rho \\ \frac{c_M^2(\mathbf{u})}{\rho} & v_{\mathrm{g}}\end{bmatrix}\frac{\partial}{\partial x}\begin{bmatrix}\rho \\ v_{\mathrm{g}}\end{bmatrix} = \begin{bmatrix}0 \\ \tilde{S}\end{bmatrix}, \begin{bmatrix}\lambda_{s1} \\ \lambda_{s2}\end{bmatrix} = \begin{bmatrix} v_{\mathrm{g}} + c_M(\mathbf{u}) \\ v_{\mathrm{g}} - c_M(\mathbf{u})\end{bmatrix}$$

- Equivalent to well known wave equation for $c_M(\mathbf{u}) \gg v_{\mathrm{g}}$.

System diagonalization

- A topside choke equation introduces an additional slow compressional pressure mode.
- Choke pressure can be derived from consideration on flow in and out and expansion of gas in the well.

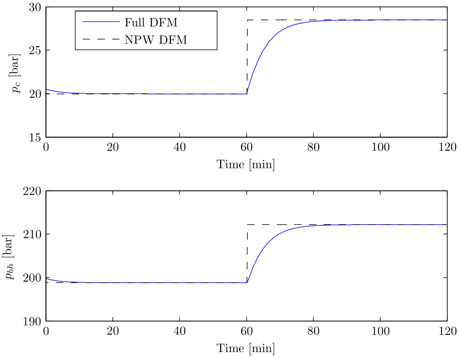


Figure: Pressure response due to change in choke opening.

Time-scale heuristic summary

10 minutes to hours: Void wave advection (movement of mass)

$$\frac{\partial \chi_\ell}{\partial t} + v_{\mathrm{g}} \frac{\partial \chi_\ell}{\partial x} = 0$$

1-10 minutes: Compressional pressure mode

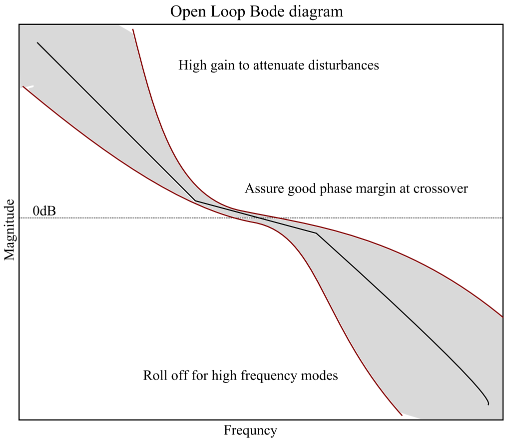
$$\frac{\partial P(x{=}L)}{\partial t} = \frac{\beta}{V}\left(q(x{=}0) - q(x{=}L) + T_{E_G}\right),$$

∼10 seconds: Distributed pressure dynamics:

$$\begin{aligned} \frac{\partial P}{\partial t} + \bar{\beta}\frac{\partial v}{\partial x} &= 0 \\ \rho\frac{\partial v}{\partial t} + \frac{\partial P}{\partial x} &= F(v) + G \end{aligned}$$

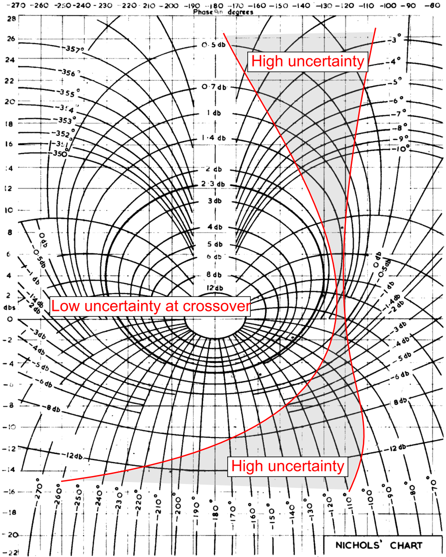
Bode Diagram

- A typical open loop bode diagram is shown below.
- We note that we can accept high uncertainties at very low and very high frequencies. But, we want low uncertainty at crossover.



Nichols chart

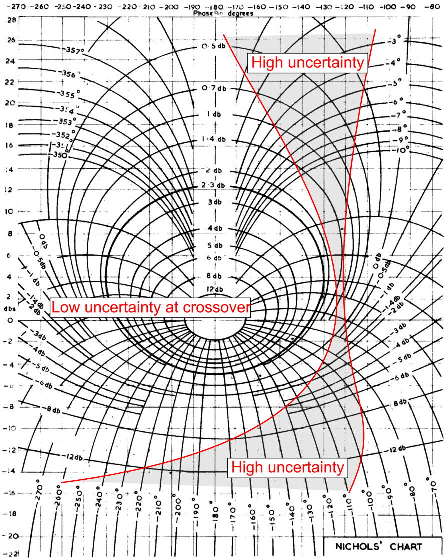
Think of a Nichols chart



- We can accept uncertainties at very low and very high frequencies.
- We just need to minimize the uncertainty around the open loop crossover frequency around $\sim 1-2$ minutes.
- I.e. discard gas dynamics and fast pressure modes.
- Only keep: slow pressure mode.

Nichols chart

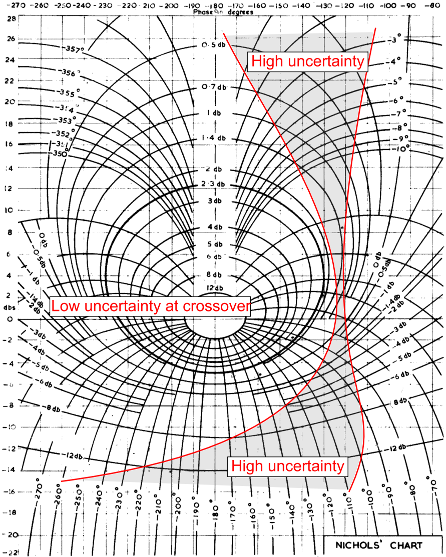
Think of a Nichols chart



- We can accept uncertainties at very low and very high frequencies.
- We just need to minimize the uncertainty around the open loop crossover frequency around $\sim 1-2$ minutes.
- I.e. discard gas dynamics and fast pressure modes.
- Only keep: slow pressure mode.

Nichols chart

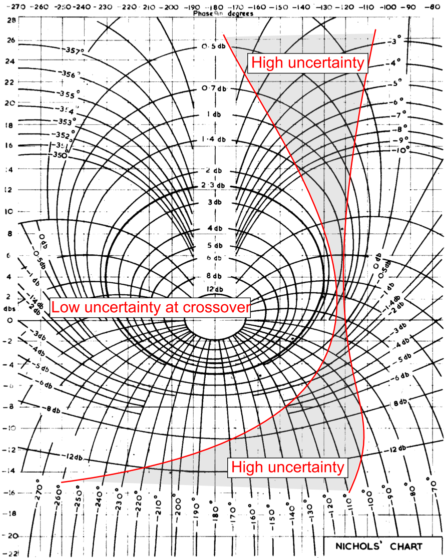
Think of a Nichols chart



- We can accept uncertainties at very low and very high frequencies.
- We just need to minimize the uncertainty around the open loop crossover frequency around $\sim 1 - 2$ minutes.
- I.e. discard gas dynamics and fast pressure modes.
- Only keep: slow pressure mode.

Nichols chart

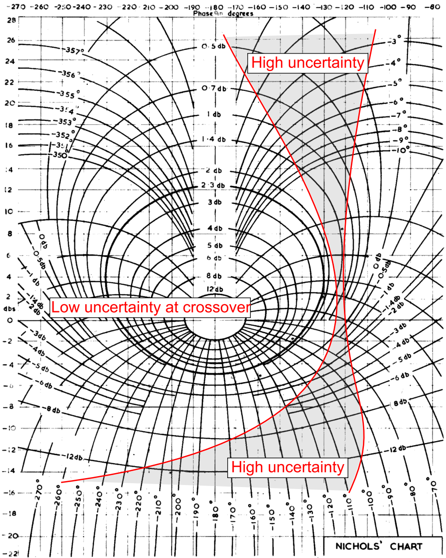
Think of a Nichols chart



- We can accept uncertainties at very low and very high frequencies.
- We just need to minimize the uncertainty around the open loop crossover frequency around $\sim 1 - 2$ minutes.
- I.e. discard gas dynamics and fast pressure modes.
- Only keep: slow pressure mode.

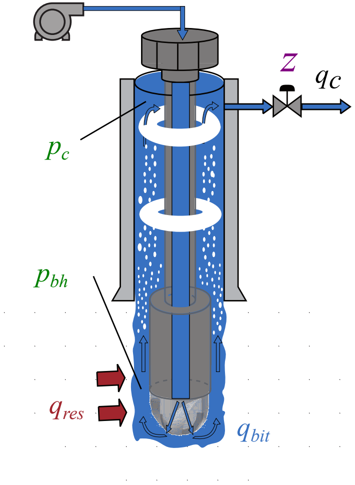
Nichols chart

Think of a Nichols chart



- We can accept uncertainties at very low and very high frequencies.
- We just need to minimize the uncertainty around the open loop crossover frequency around $\sim 1-2$ minutes.
- I.e. discard gas dynamics and fast pressure modes.
- Only keep: slow pressure mode.

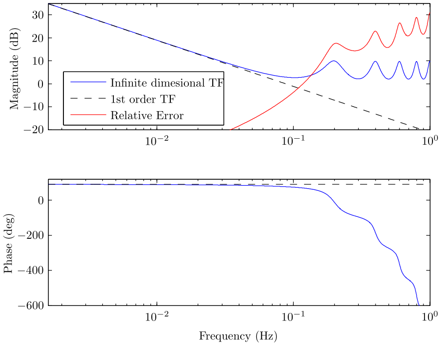
Approximated Pressure Dynamics - one phase



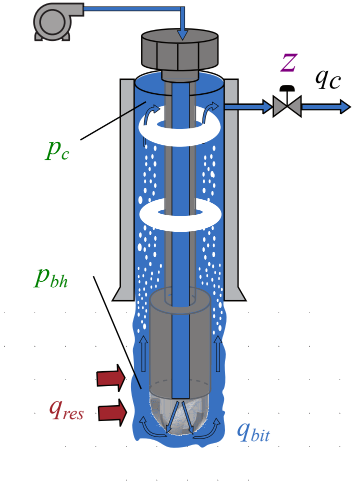
Full infinite dimensional description:

$$\frac{P_{bh}}{Q_c}(s) = \frac{Z_c(s)}{\tanh \Gamma(s)}$$

$$\lim_{s \to 0} \frac{P_{bh}}{Q_c}(s) \underset{\tanh \Gamma \to \Gamma}{\rightarrow} \frac{Z_c(s)}{\Gamma(s)} = \frac{\beta}{sV}$$

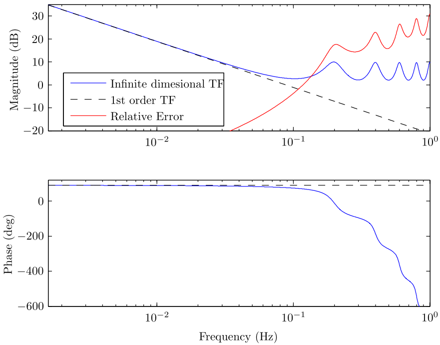


Approximated Pressure Dynamics - one phase

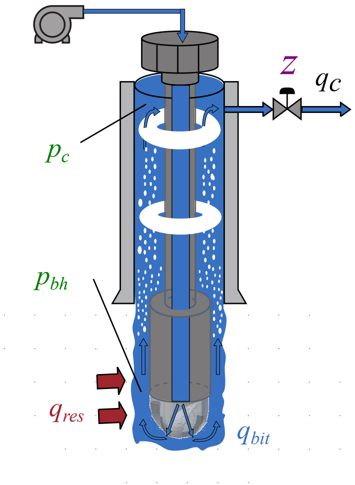


Lumped approximation

$$\dot{p}_{bh} \approx \frac{\beta}{V}(q_{in} - q_{out})$$



Approximated Pressure Dynamics - two phase



First order lumped approximation

$$\dot{p}_{bh} \approx \frac{\bar{\beta}(t)}{V}(q_{bit} - q_c + w(t))$$

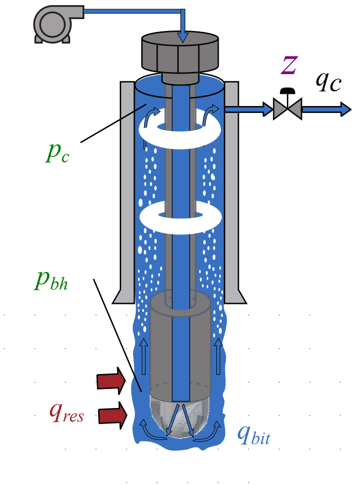
$$\bar{\beta}(t) = \frac{L}{\int_0^L \left[\frac{\alpha_g(x,t)}{p(x,t)} + \frac{1-\alpha_g(x,t)}{\beta_L}\right] \mathrm{d}x}.$$

with *slow* changes in hydrostatic pressure $w(t)$ and bulk modulus $\bar{\beta}(t)$.

Key uncertainties:

- Uncertain gas profile $\alpha_g(x,t)$
- High frequency uncertainty due to model reduction

Approximated Pressure Dynamics - two phase



First order lumped approximation

$$\dot{p}_{bh} \approx \frac{\bar{\beta}(t)}{V}(q_{bit} - q_c + w(t))$$

$$\bar{\beta}(t) = \frac{L}{\int_0^L \left[\frac{\alpha_g(x,t)}{p(x,t)} + \frac{1-\alpha_g(x,t)}{\beta_L}\right] \mathrm{d}x}.$$

with *slow* changes in hydrostatic pressure $w(t)$ and bulk modulus $\bar{\beta}(t)$.

Key uncertainties:

- Uncertain gas profile $\alpha_g(x, t)$
- High frequency uncertainty due to model reduction

Linearize choke equation

Flow out given as $q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}}$.

- define static actuation mapping $z(u) = C_v^{-1}\left(q_{bit}\frac{\sqrt{\rho_\ell}}{\sqrt{u}}\right)$

$$q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}} = q_{bit}\frac{\sqrt{p_c - p_{c0}}}{\sqrt{u}}$$

- Linearize choke equation (around operating point)

$$\tilde{q}_c \approx K_p\tilde{p}_c - K_p\tilde{u}$$

With K_p *known* and dependent on $\bar{p}_c$ and $\bar{u}$.

- Linearized perturbation dynamics, time constant $\tau(t)$:

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\left(-\tilde{p}_{bh} + \tilde{u} + w\right)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(\alpha_g(t))}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{\bar{u}(t)}.$$

Linearize choke equation

Flow out given as $q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}}$.

- define static actuation mapping $z(u) = C_v^{-1}\left(q_{bit}\frac{\sqrt{\rho_\ell}}{\sqrt{u}}\right)$

$$q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}} = q_{bit}\frac{\sqrt{p_c - p_{c0}}}{\sqrt{u}}$$

- Linearize choke equation (around operating point)

$$\tilde{q}_c \approx K_p\tilde{p}_c - K_p\tilde{u}$$

With K_p *known* and dependent on $\bar{p}_c$ and $\bar{u}$.

- Linearized perturbation dynamics, time constant $\tau(t)$:

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\left(-\tilde{p}_{bh} + \tilde{u} + w\right)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(\alpha_g(t))}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{\bar{u}(t)}.$$

Linearize choke equation

Flow out given as $q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}}$.

- define static actuation mapping $z(u) = C_v^{-1}\left(q_{bit}\frac{\sqrt{\rho_\ell}}{\sqrt{u}}\right)$

$$q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}} = q_{bit}\frac{\sqrt{p_c - p_{c0}}}{\sqrt{u}}$$

- Linearize choke equation (around operating point)

$$\tilde{q}_c \approx K_p\tilde{p}_c - K_p\tilde{u}$$

With K_p *known* and dependent on $\bar{p}_c$ and $\bar{u}$.

- Linearized perturbation dynamics, time constant $\tau(t)$:

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\big(-\tilde{p}_{bh} + \tilde{u} + w\big)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(\alpha_g(t))}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{\bar{u}(t)}.$$

Linearize choke equation

Flow out given as $q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}}$.

- define static actuation mapping $z(u) = C_v^{-1}\left(q_{bit}\frac{\sqrt{\rho_\ell}}{\sqrt{u}}\right)$

$$q_c = \frac{C_v(z)}{\sqrt{\rho_\ell}}\sqrt{p_c - p_{c0}} = q_{bit}\frac{\sqrt{p_c - p_{c0}}}{\sqrt{u}}$$

- Linearize choke equation (around operating point)

$$\tilde{q}_c \approx K_p\tilde{p}_c - K_p\tilde{u}$$

  With K_p *known* and dependent on $\bar{p}_c$ and $\bar{u}$.

- Linearized perturbation dynamics, time constant $\tau(t)$:

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\big(-\tilde{p}_{bh} + \tilde{u} + w\big)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(\alpha_g(t))}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{\bar{u}(t)}.$$

Slow mode time constant product of two parts

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\big(-\tilde{p}_{bh} + \tilde{u} + w\big)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(t)}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{u(t)}.$$

with time-varying $K_p(t)$ known and $\bar{\beta}(t)$ uncertain.

- Given estimate $\hat{\tau}(t)$
- Define a robustness coefficient r giving relative uncertainty in $\tau(t)$:

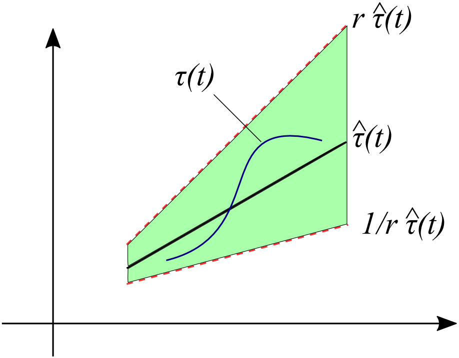
$$\tau(t) \in [r\hat{\tau}(t), \frac{1}{r}\hat{\tau}(t)]$$

Slow mode time constant product of two parts

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\big(-\tilde{p}_{bh} + \tilde{u} + w\big)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(t)}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{u(t)}.$$

with time-varying $K_p(t)$ known and $\bar{\beta}(t)$ uncertain.



- Given estimate $\hat{\tau}(t)$
- Define a robustness coefficient r giving relative uncertainty in $\tau(t)$:

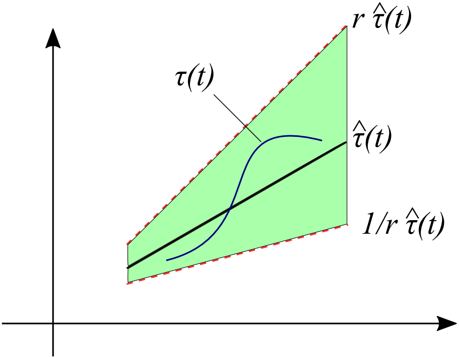
$$\tau(t) \in [r\hat{\tau}(t), \frac{1}{r}\hat{\tau}(t)]$$

Slow mode time constant product of two parts

$$\dot{\tilde{p}}_{bh}(t) \approx \frac{1}{\tau(t)}\big(-\tilde{p}_{bh} + \tilde{u} + w\big)$$

$$\tau(t) = \frac{V}{K_p(t)\bar{\beta}(t)}, \quad K_p(t) = \frac{\bar{q}_{bh}}{2C_K(t)}\frac{1}{u(t)}.$$

with time-varying $K_p(t)$ known and $\bar{\beta}(t)$ uncertain.



- Given estimate $\hat{\tau}(t)$
- Define a robustness coefficient r giving relative uncertainty in $\tau(t)$:

$$\tau(t) \in [r\hat{\tau}(t), \frac{1}{r}\hat{\tau}(t)]$$

Minimize control error subject to robustness to uncertainties:

- Time constant $\tau(t)$ with coefficient r
- High frequency dynamics p with coefficient $\Delta\tau$

Control problem formulation

Find a controller mapping from $\tilde{p}_{bh}$ to $\tilde{u}$ that robustly minimizes the $\mathbf{L}_2$ gain

$$\sup_{\|w\|_2 \neq 0} \frac{\|I_e\|_2}{\|w\|_2},$$

subject to

$$\begin{aligned}
\dot{\tilde{p}}_{bh} &= \frac{1}{\tau(t)}\left(-\tilde{p}_{bh} + \tilde{u} + w + \tau_\Delta p\right), \\
\dot{I}_e &= \tilde{p}_{bh}, \\
p &= \Delta(t)\dot{\tilde{u}}, \quad \|\Delta(t)\| \le 1, \quad \tau(t) \in [r\hat{\tau}(t), \frac{1}{r}\hat{\tau}(t)].
\end{aligned}$$

Minimize control error subject to robustness to uncertainties:

- Time constant $\tau(t)$ with coefficient r
- High frequency dynamics p with coefficient $\Delta\tau$

Control problem formulation

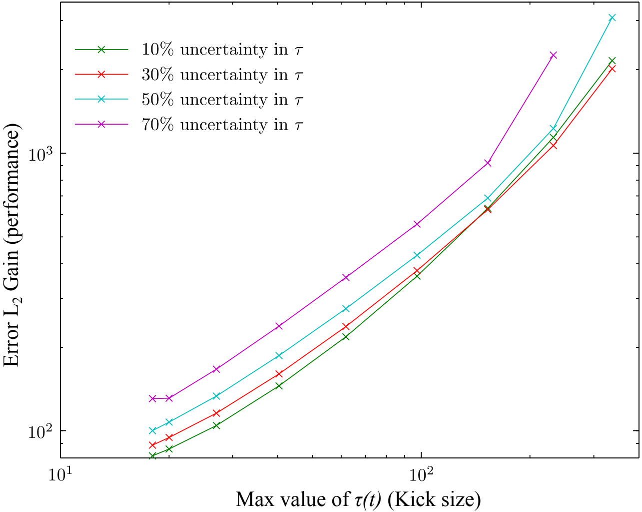
Find a controller mapping from $\tilde{p}_{bh}$ to $\tilde{u}$ that robustly minimizes the $\mathbf{L}_2$ gain

$$\sup_{\|w\|_2 \neq 0} \frac{\|I_e\|_2}{\|w\|_2},$$

subject to

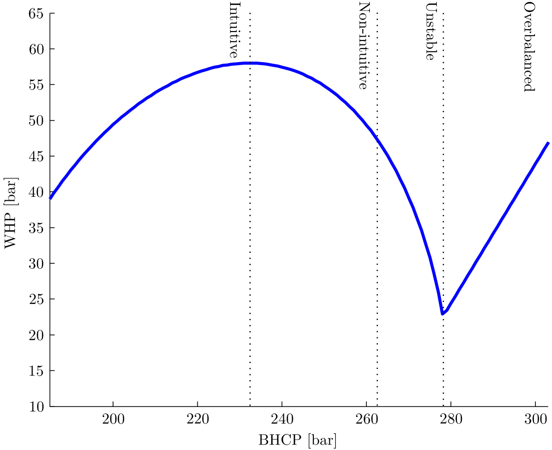
$$\begin{aligned}
\dot{\tilde{p}}_{bh} &= \frac{1}{\tau(t)} \left(-\tilde{p}_{bh} + \tilde{u} + w + \tau_\Delta p\right), \\
\dot{I}_e &= \tilde{p}_{bh}, \\
p &= \Delta(t)\dot{\tilde{u}}, \quad \|\Delta(t)\| \leq 1, \quad \tau(t) \in [r\hat{\tau}(t), \frac{1}{r}\hat{\tau}(t)].
\end{aligned}$$

Performance / robustness trade-off



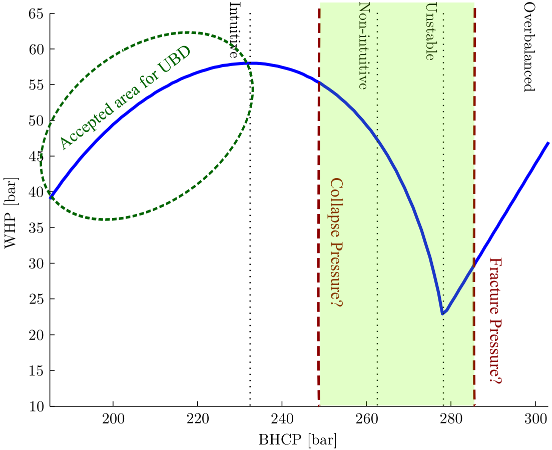
Summary

- Classification of UBD operating regimes
-



Summary

- Classification of UBD operating regimes
- Strong case for automatic control



Summary

- Models for control and estimation should have the right trade-off between complexity and fidelity.
- Capture the dominating dynamics for the given application.

Time scale heuristic

Time-scale	Dominating dynamics
$\sim$ 10 seconds	Distributed pressure dynamics
$\sim$ 1—10 minutes	Slow compression pressure mode
$\sim$ 10 minutes to hours	Void wave advection

Summary

- Models for control and estimation should have the right trade-off between complexity and fidelity.
- Capture the dominating dynamics for the given application.

Time scale heuristic

Time-scale	Dominating dynamics
$\sim$ 10 seconds	Distributed pressure dynamics
$\sim$ 1—10 minutes	Slow compression pressure mode
$\sim$ 10 minutes to hours	Void wave advection

Summary

- Models for control and estimation should have the right trade-off between complexity and fidelity.
- Capture the dominating dynamics for the given application.

Time scale heuristic

Time-scale	**Dominating dynamics**
$\sim$ 10 seconds	Distributed pressure dynamics
$\sim$ 1–10 minutes	Slow compression pressure mode
$\sim$ 10 minutes to hours	Void wave advection

Publications

Journal papers, Published

A1 U. J. F. Aarsnes, M. S. Gleditsch, O. M. Aamo, and A. Pavlov, “Modeling and Avoidance of Heave-Induced Resonances in Offshore Drilling,” *SPE Drill. Complet.*, vol. 29, no. 04, pp. 454-464, Dec. 2014.

A2 U. J. F. Aarsnes, F. Di Meglio, R. Graham, and O. M. Aamo, “A methodology for classifying operating regimes in underbalanced drilling operations,” *SPE J.*, 21(02), pp. 243433, Apr. 2016.

A3 U. J. F. Aarsnes and O. M. Aamo, “Linear stability analysis of self-excited vibrations in drilling using an infinite dimensional model,” *J. Sound Vib.*, vol. 360, pp. 239259, Jan. 2016.

A4 U. J. F. Aarsnes, A. Ambrus, F. Di Meglio, A. K. Vajargah, O. M. Aamo, and E. Van Oort, “A Simplified Two-Phase Flow Model Using a Quasi-Equilibrium Momentum Balance,” *Int. J. Multiph. flow*, 83(July), pp. 77-85, Jul. 2016.

A5 A. Ambrus, U. J. F. Aarsnes, A. Karimi, B. Akbari, E. van Oort and O. M. Aamo, “Real-Time Estimation of Reservoir Influx Rate and Pore Pressure Using a Simplified Transient Two-Phase Flow Model,” *J. Nat. Gas Sci. Eng.*, 32, 439-452.

Journal papers, In review

A6 U. J. F. Aarsnes, T. Flåtten, and O. M. Aamo, “Models of gas-liquid two-phase flow in drilling for control and estimation applications,” In review.

A7 U. J. F. Aarsnes, B. Açıkmeşe, A. Ambrus and O. M. Aamo, “Robust Controller Design for Automated Kick Handling in Managed Pressure Drilling,” In review.

A8 A. Nikoofard, U. J. F. Aarsnes, T. A. Johansen, and G.O. Kaasa, “State and Parameter Estimation of a Drift-Flux Model for Under Balanced Drilling Operations”. In review.

Publications (cont.)

Conference papers

B1 U. J. F. Aarsnes, O. M. Aamo, and A. Pavlov, "Quantifying Error Introduced by Finite Order Discretization of a Hydraulic Well Model," in *Australian Control Conference*, 2012, pp. 54–59.

B2 U. J. F. Aarsnes, O. M. Aamo, E. Hauge, and A. Pavlov, "Limits of Controller Performance in the Heave Disturbance Attenuation Problem," in *European Control Conference (ECC)*, 2013, pp. 1070–1076.

B3 U. J. F. Aarsnes, F. Di Meglio, S. Evje, and O. M. Aamo, "Control-Oriented Drift-Flux Modeling of Single and Two-Phase Flow for Drilling," in *ASME Dynamic Systems and Control Conference*, 2014.

B4 U. J. F. Aarsnes, A. Ambrus, A. Karimi Vajargah, O. M. Aamo, and E. van Oort, "A simplified gas-liquid flow model for kick mitigation and control during drilling operations," in *Proceedings of the ASME 2015 Dynamic Systems and Control Conference*, 2015.

B5 F. Di Meglio and U. J. F. Aarsnes, "A distributed parameter systems view of control problems in drilling," in *2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, 2015.

B6 F. Di Meglio, D. Bresch-Pietri, and U. J. F. Aarsnes, "An Adaptive Observer for Hyperbolic Systems with Application to UnderBalanced Drilling," in *IFAC World Congress 2014*, South Africa, 2014, pp. 11391–11397.

B7 A. Nikoofard, U. J. F. Aarsnes, T. A. Johansen, and G.-O. Kaasa, "Estimation of States and Parameters of a Drift-Flux Model with Unscented Kalman Filter," in *2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, 2015.

Publications without peer review

C1 U. J. F. Aarsnes, F. Di Meglio, O. M. Aamo, and G.-O. Kaasa, "Fit-for-Purpose Modeling for Automation of Underbalanced Drilling Operations," in *SPE/IADC Managed Pressure Drilling & Underbalanced Operations Conference & Exhibition*, 2014.

C2 U. J. F. Aarsnes, H. Mahdianfar, O. M. Aamo and A. Pavlov. " Rejection of Heave-Induced Pressure Oscillations in Managed Pressure Drilling," presented at the *Colloquium on Nonlinear Dynamics and Control of Deep Drilling Systems*, Minneapolis, Minnesota, May 2014. (Invited Paper).

C4 A. Ambrus, U. J. F. Aarsnes, A. Karimi Vajargah, B. Akbari and E. van Oort, "A Simplified Transient Multi-Phase Model for Automated Well Control Applications," in *9th International Petroleum Conf. (IPTC)*, 2015.

References I

Baer, M. and Nunziato, J. (1986). A two-phase mixture theory for the deflagration-to-detonation transition (ddt) in reactive granular materials.

Evje, S. and Wen, H. (2015). Global Solutions of a Viscous Gas-Liquid Model with Unequal Fluid Velocities in a Closed Conduit. *SIAM Journal on Mathematical Analysis*, 47(1):381–406.

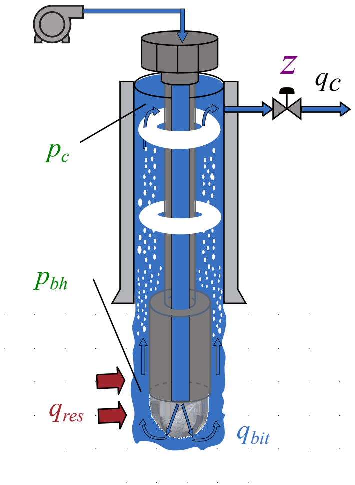
Gavrilyuk, S. and Fabre, J. (1996). Lagrangian coordinates for a drift-flux model of a gas-liquid mixture. *International journal of multiphase flow*, 22(3):453–460.

Lage, A., Fjelde, K., and Time, R. (2000). Underbalanced Drilling Dynamics: Two-Phase Flow Modeling and Experiments. In *IADC/SPE Asia Pacific Drilling Technology*, volume 8, pages 61–70, Kuala Lumpur, Malaysia. Society of Petroleum Engineers.

Linga, G. (2016). A hierarchy of non-equilibrium two-phase flow models. In review.

Zuber, N. and Findlay, J. A. (1965). Average Volumetric Concentration in Two-Phase Flow Systems. *Journal of Heat Transfer*, 87(4):453.

Model based influx estimation

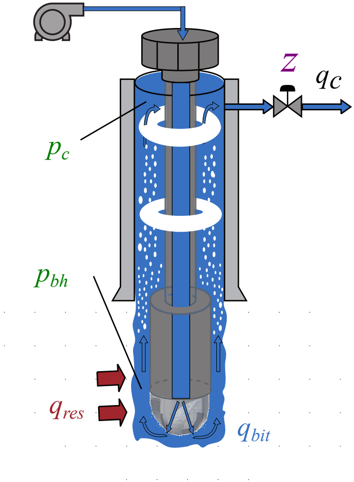


- Detecting influx from reservoir usually done by

$$q_{res} \approx q_c - q_{bit}$$

- Does not account for changes in pressure and gas expansion.
- Improved estimate using measured variables p_c, q_c, q_{bit} to obtain unmeasured quantity q_{res}
- Need simple model which allows for "inverting" the dynamics.

Model based influx estimation

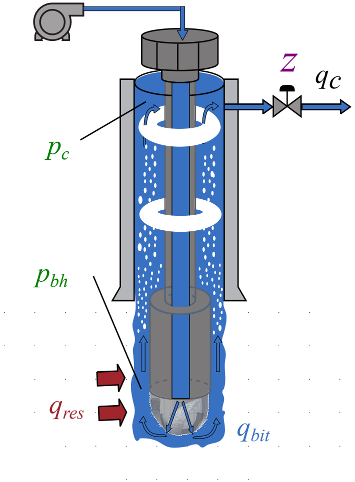


- Detecting influx from reservoir usually done by

$$q_{res} \approx q_c - q_{bit}$$

- Does not account for changes in pressure and gas expansion.
- Improved estimate using measured variables p_c, q_c, q_{bit} to obtain unmeasured quantity q_{res}
- Need simple model which allows for "inverting" the dynamics.

Model based influx estimation

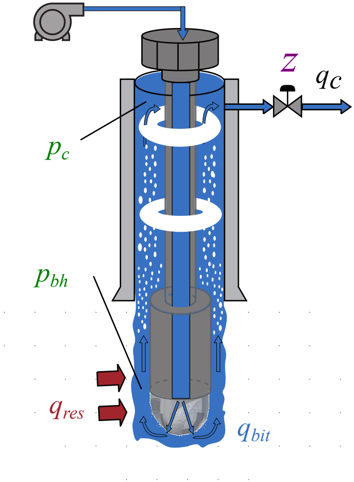


- Detecting influx from reservoir usually done by

$$q_{res} \approx q_c - q_{bit}$$

- Does not account for changes in pressure and gas expansion.
- Improved estimate using measured variables p_c, q_c, q_{bit} to obtain unmeasured quantity q_{res}
- Need simple model which allows for "inverting" the dynamics.

Model based influx estimation

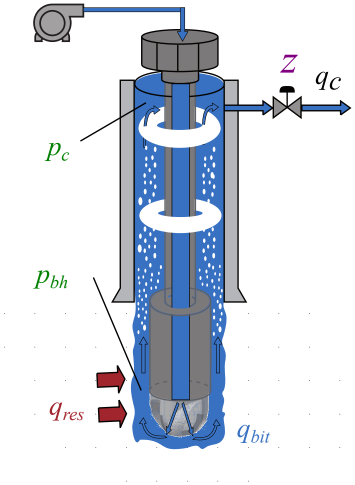


- Detecting influx from reservoir usually done by

$$q_{res} \approx q_c - q_{bit}$$

- Does not account for changes in pressure and gas expansion.
- Improved estimate using measured variables p_c, q_c, q_{bit} to obtain unmeasured quantity q_{res}
- Need simple model which allows for "inverting" the dynamics.

Approximated Pressure Dynamics



Lumped pressure dynamics

$$\dot{p}_c = \frac{\bar{\beta}}{V}(q_{bit} + q_{res} - q_c + T_{XE})$$

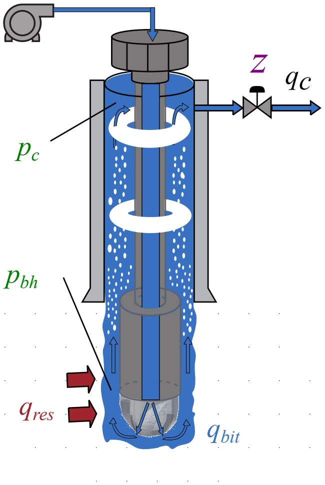
Simplified dynamics of void fraction α_g propagation

$$\frac{\partial \alpha_g}{\partial t} + v_g \frac{\partial \alpha_g}{\partial x} = E_g(\alpha_g)$$

$$\alpha_g(x=0) = \frac{q_{res}}{C_0(q_{res} + q_{bit}) + Av_\infty}$$

$$T_{XE} = A \int_0^L E_g \mathrm{d}x$$

Approximated Pressure Dynamics



Lumped pressure dynamics

$$\dot{p}_c = \frac{\bar{\beta}}{V}(q_{bit} + q_{res} - q_c + T_{XE})$$

Simplified dynamics of void fraction α_g propagation

$$\frac{\partial \alpha_g}{\partial t} + v_g \frac{\partial \alpha_g}{\partial x} = E_g(\alpha_g)$$

$$\alpha_g(x=0) = \frac{q_{res}}{C_0(q_{res} + q_{bit}) + A v_\infty}$$

$$T_{XE} = A \int_0^L E_g \mathrm{d}x$$

Estimation formulation

- Use lumped pressure dynamics

$$\dot{p}_c = \frac{\beta}{V}(q_{bit} + q_{res} - q_c + T_{XE})$$
$$\implies \frac{\beta}{V} q_{res} = \dot{p}_c - \frac{\bar{\beta}}{V}(q_{bit} - q_c + T_{XE})$$

- Apply low-pass filter $\frac{1}{\tau s+1}$, estimate $\hat{\theta} = \frac{1}{\tau s+1}\frac{\bar{\beta}}{V} q_{res}$:

$$\hat{\theta} = \frac{s}{\tau s + 1}[p_c] - \frac{1}{\tau s + 1}\left[\frac{\hat{\beta}}{V}(q_{bit} - q_c + \hat{T}_{XE})\right]$$

with values measured and computed

- $\hat{\theta}$ used to detect kick and estimate IPR and p_{res}:

$$q_{res} = J \max(p_{res} - p_{bh}).$$

Estimation formulation

- Use lumped pressure dynamics

$$\dot{p}_c = \frac{\beta}{V}(q_{bit} + q_{res} - q_c + T_{XE})$$
$$\implies \frac{\beta}{V} q_{res} = \dot{p}_c - \frac{\bar{\beta}}{V}(q_{bit} - q_c + T_{XE})$$

- Apply low-pass filter $\frac{1}{\tau s+1}$, estimate $\hat{\theta} = \frac{1}{\tau s+1}\frac{\bar{\beta}}{V} q_{res}$:

$$\hat{\theta} = \frac{s}{\tau s + 1}[p_c] - \frac{1}{\tau s + 1}\left[\frac{\hat{\beta}}{V}(q_{bit} - q_c + \hat{T}_{XE})\right]$$

with values measured and computed

- $\hat{\theta}$ used to detect kick and estimate IPR and p_{res}:

$$q_{res} = J \max(p_{res} - p_{bh}).$$

Estimation formulation

- Use lumped pressure dynamics

$$\dot{p}_c = \frac{\beta}{V}(q_{bit} + q_{res} - q_c + T_{XE})$$
$$\implies \frac{\beta}{V} q_{res} = \dot{p}_c - \frac{\bar{\beta}}{V}(q_{bit} - q_c + T_{XE})$$

- Apply low-pass filter $\frac{1}{\tau s+1}$, estimate $\hat{\theta} = \frac{1}{\tau s+1}\frac{\bar{\beta}}{V} q_{res}$:

$$\hat{\theta} = \frac{s}{\tau s + 1}[p_c] - \frac{1}{\tau s + 1}\left[\frac{\hat{\beta}}{V}(q_{bit} - q_c + \hat{T}_{XE})\right]$$

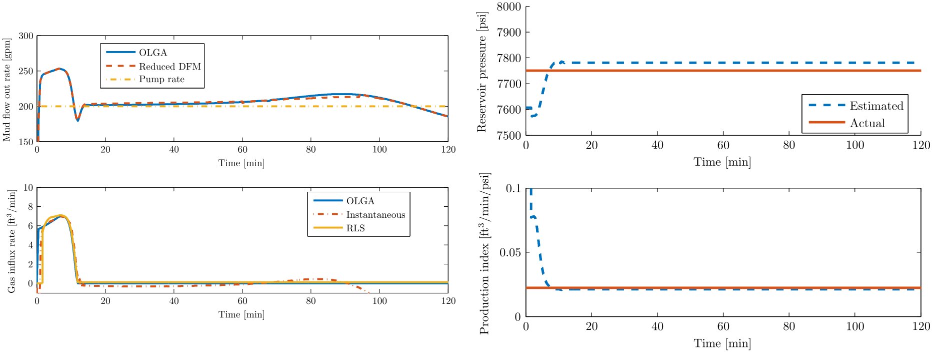
with values measured and computed

- $\hat{\theta}$ used to detect kick and estimate *IPR* and p_{res}:

$$q_{res} = J \max(p_{res} - p_{bh}).$$

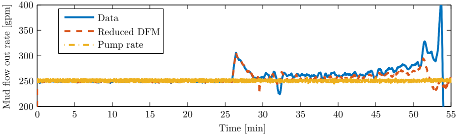
OLGA simulated kick

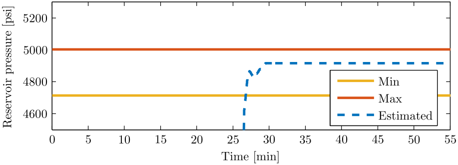
Performance of reservoir estimation on simulated kick

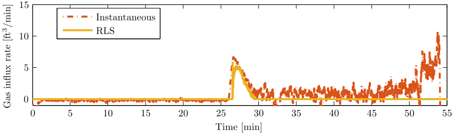


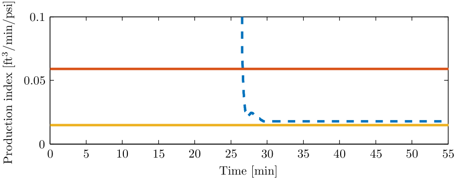
Application to field data

Application to estimation of kick dynamically handled by Microflux.









- Minimum and maximum values discerned from field logs.
- Initial estimation gives reasonable results.
- Estimation error deviates over time due to lack of feedback.

Derivation of reduced DFM

- Liquid mass conservation

$$\frac{\partial\left[\alpha_L\rho_L\right]}{\partial t}+\frac{\partial\left[\alpha_L\rho_L v_L\right]}{\partial x}=0, \quad \rho_L\approx \text{const.}$$
$$\implies \frac{\partial\alpha_L}{\partial t}+\frac{\partial\alpha_L}{\partial x}v_G+\alpha_L\frac{\partial v_G}{\partial x}=0$$

- Gives conservation of void fraction

$$\implies \frac{\partial\alpha_G}{\partial t}+v_G\frac{\partial\alpha_G}{\partial x}=E_G$$

where $E_G\equiv\alpha_L\frac{\partial v_G}{\partial x}$ is the local gas expansion.

- Similarly we obtain:

$$\frac{\partial v_G}{\partial x}=\frac{E_G}{\alpha_G}$$

Pressure dynamics

- 1-st order pressure dynamics

$$\frac{\partial p_c}{\partial t} = \frac{\beta_L}{V}\left(q_L + q_G + T_{E_G} - q_C\right)$$

- Where the total gas expansion is given as

$$T_{E_G} = A\int_0^L E_G(x)\mathrm{d}x \tag{8}$$

- And the distributed pressure from the steady momentum equation

$$P(x) = p_c + \int_L^x G(x) + F(x)\mathrm{d}x$$

Effective bulk modulus

- Returning to the local gas expansion, use the approximation $\frac{\partial P}{\partial t} \approx \frac{\partial p_c}{\partial t}$:

$$\frac{T_{E_G}}{A} = \int_0^L -\frac{\alpha_G \alpha_L}{P}\left(\frac{\partial P}{\partial t} + v_G \frac{\partial P}{\partial x}\right)\mathrm{d}x$$
$$= -\frac{\partial p_c}{\partial t}\int_0^L \frac{\alpha_G \alpha_L}{P}\mathrm{d}x + \int_0^L \frac{\alpha_G \alpha_L}{P}\Big(G(x) + F(x)\Big)\mathrm{d}x$$

- Thus the pressure dynamics rewrite

$$\frac{\partial p_c}{\partial t} = \frac{\beta_L}{V}\left(q_L + q_G + T_{E_G} - q_C\right)$$
$$= \frac{\beta_L}{1 + \beta_L \frac{A}{V}\int_0^L \frac{\alpha_G \alpha_L}{P}\mathrm{d}x}\left(q_L + q_G + T_{XE} - q_C\right)$$

Effective bulk modulus

- Returning to the local gas expansion, use the approximation $\frac{\partial P}{\partial t} \approx \frac{\partial p_c}{\partial t}$:

$$\frac{T_{E_G}}{A} = \int_0^L -\frac{\alpha_G \alpha_L}{P} \left(\frac{\partial P}{\partial t} + v_G \frac{\partial P}{\partial x} \right) \mathrm{d}x$$
$$= -\frac{\partial p_c}{\partial t} \int_0^L \frac{\alpha_G \alpha_L}{P} \mathrm{d}x + \int_0^L \frac{\alpha_G \alpha_L}{P} \Big(G(x) + F(x) \Big) \mathrm{d}x$$

- Thus the pressure dynamics rewrite

$$\frac{\partial p_c}{\partial t} = \frac{\beta_L}{V} \left(q_L + q_G + T_{E_G} - q_C \right)$$
$$= \frac{\beta_L}{1 + \beta_L \frac{A}{V} \int_0^L \frac{\alpha_G \alpha_L}{P} \mathrm{d}x} \left(q_L + q_G + T_{XE} - q_C \right)$$